2.- $U \equiv \{<(1,a,1),(1,1,1),(0,0,a)>\}$

$$\begin{pmatrix} 1 & a & 1 \\ 1 & 1 & 1 \\ 0 & 0 & a \end{pmatrix} \rightarrow \begin{pmatrix} 0 & a-1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & a \end{pmatrix}$$

- $a = 0$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$U \equiv \{<(0,-1,0),(1,0,1)>\}$

$U \equiv \begin{cases} x = \lambda \\ y = -\mu \\ z = \lambda \end{cases} \qquad U \equiv \{ x - z = 0$

- $a = 1$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U \equiv \{<(1,1,0),(0,0,1)>\}$$

$U \equiv \begin{cases} x = \lambda \\ y = \lambda \\ z = \mu \end{cases}$

3.- $\{<(2,5,3),(0,-1,-1)>\} = \{<(4,9,5),(2,7,5)>\}$

$$\begin{pmatrix} 2 & 5 & 3 \\ 0 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad U_1 = \{<(2,5,3),(0,-1,-1)>\} =$$

$$\begin{pmatrix} 4 & 9 & 5 \\ 2 & 7 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad = \{<(1,1,0),(0,1,1)>\} =$$

$= \{<(4,9,5),(2,7,5)>\}$

4:

• $\begin{pmatrix} 5 & 8 \\ 1 & -1 \end{pmatrix} = A$

$dA = -13$

$A^{-1} = \begin{pmatrix} \frac{1}{13} & \frac{8}{13} \\ \frac{1}{13} & \frac{-5}{13} \end{pmatrix}$

$A^t = \begin{pmatrix} 5 & 1 \\ 8 & -1 \end{pmatrix}$

$adj(A^t) = \begin{pmatrix} -1 & -8 \\ -1 & 5 \end{pmatrix}$

• $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 3 & 0 \end{pmatrix} = B$

$\det B = -1$

$B^t = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

$adj\ B^t = \begin{pmatrix} 3 & 3 & -2 \\ -2 & -2 & 1 \\ +1 & 2 & -1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} -3 & -3 & 2 \\ +2 & +2 & -1 \\ -1 & -2 & 1 \end{pmatrix}$

Algebra Lineal

Ejercicios propuestos

5.

$$\left.\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ -x_1 + 2x_2 + 2x_3 + 3x_4 &= 4 \\ 2x_1 - x_2 &= -2 \\ 3x_1 + 3x_2 - 2x_3 + x_4 &= -2 \end{aligned}\right\}$$

a)

$$\left(\begin{array}{cccc:c} 1 & -2 & 2 & 1 & 0 \\ -1 & 2 & 2 & 3 & 4 \\ 2 & -1 & & & -2 \\ 3 & 3 & -2 & 1 & -2 \end{array}\right) \xrightarrow[\substack{-2F_1+F_3 \to F_3 \\ -3F_1+F_4 \to F_4}]{F_2+F_1 \to F_2} \left(\begin{array}{cccc:c} 1 & -2 & 2 & 1 & 0 \\ 0 & 0 & 4 & 4 & 4 \\ 0 & 3 & -4 & -2 & -2 \\ 0 & 9 & -8 & -2 & -2 \end{array}\right) \begin{array}{l} F_2 \to F_3 \\ F_2+F_3 \to F_2 \\ \to \\ 2F_3+F_4 \to F_4 \\ F_3/4 \to F_3 \end{array}$$

$$\left(\begin{array}{cccc:c} 1 & -2 & 2 & 1 & 0 \\ 0 & 3 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 9 & 0 & 6 & 6 \end{array}\right) \xrightarrow{F_4-3F_2 \to F_4} \left(\begin{array}{cccc:c} 1 & -2 & 2 & 1 & 0 \\ 0 & 3 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ 3x_2 + 2x_4 &= 2 \\ x_3 + x_4 &= 1 \end{aligned}\right\} \qquad \left\{\begin{aligned} x_1 &= (-2-\lambda)/3 \\ x_2 &= (2-2\lambda)/3 \\ x_3 &= 1-\lambda \\ x_4 &= \lambda \end{aligned}\right. \qquad (-1/3, -2/3, -1, 1)$$

b)

6.

$$\left.\begin{aligned} x+y+z&=1\\ 2x+2y+(1-a^2)z&=2a\\ x+y+a^2z&=-1\end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 2 & 2 & (1-a^2) & 2a\\ 1 & 1 & a^2 & -1\end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 0 & 0 & -a^2-1 & 2a-2\\ 0 & 0 & a^2-1 & -2\end{array}\right) \to$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 0 & 0 & -1-a^2 & 2a-2\\ 0 & 0 & -2 & 2a-4\end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 0 & 0 & a^2+1 & 2-2a\\ 0 & 0 & 1 & 2-a\end{array}\right) \to$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 0 & 0 & a^2 & -a\\ 0 & 0 & 1 & 2-a\end{array}\right)$$

<u>Caso 1</u>

$a=0$ ó $a=1+\sqrt{2}$ ó $a=1-\sqrt{2}$

- $a=0$

SCI

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\end{array}\right)$$

- $a=1+\sqrt{2}$

SCI

- $a=1-\sqrt{2}$

SCI

<u>Caso 2</u>

$a\neq 0$ y $a\neq 1+\sqrt{2}$ y $a\neq 1-\sqrt{2}$

S. I

7:

$$\begin{cases} x_1 - 4y + 3z = a \\ x + 2y + 7z = b \\ 2x - 2y + 10z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & 3 & a \\ 1 & 2 & 7 & b \\ 2 & -2 & 10 & 0 \end{array}\right) \longrightarrow \begin{pmatrix} 1 & -4 & 3 & a \\ 1 & 2 & 7 & b \\ 0 & 0 & 0 & -a-b \end{pmatrix}$$

$$\begin{pmatrix} 1 & -4 & 3 & a \\ 0 & 6 & 4 & b-a \\ 0 & 0 & 0 & -a-b \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -4 & 3 & a \\ 0 & 3 & 2 & -a \\ 0 & 0 & 0 & -a-b \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 5 & 0 \\ 0 & 3 & 2 & -a \\ 0 & 0 & 0 & +a+b \end{pmatrix}$$

- Si $a = -b \iff$ SCI
- Si $a \neq -b \iff$ SI

8: U y V de $\mathbb{R}^3$

$U \equiv \{ x_1 - 2x_2 + x_3 = 0$, $V = \begin{cases} x_1 = 2t \\ x_2 = t \\ x_3 = 3\lambda \end{cases}$

$V : \{ x - 2y = 0$

$U \equiv \begin{cases} x_1 = 2t - \lambda \\ x_2 = t \\ x_3 = \lambda \end{cases}$

$$U \cap V \equiv \begin{cases} x - 2y = 0 \\ x - 2y + z = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U \cap V \equiv \begin{cases} x - 2y = 0 \\ z = 0 \end{cases}$$ Ecuación cartesiana

$$U \cap V \equiv \begin{cases} x = 2\lambda \\ y = \lambda \\ z = 0 \end{cases}$$ Ecuación paramétrica

Y la base $(2, 1, 0)$ $\quad \dim(U \cap V) = 1$

$$U + V = \mathcal{L}\{(2,1,0), (0,0,3), (-1,0,1)\}$$

$$U + V = \begin{cases} x = 2\cancel{\lambda} - \mu \\ \cancel{y = \lambda} \\ \cancel{z} = 3t + \mu \end{cases}$$? $\quad \mathrm{Dim}(U+V) = 3 \Rightarrow U + V = \mathbb{R}^3$

9: $U = \{ <(1,0,1,1),(1,-1,-1,0),(0,1,2,1)> \}$

$V = \{(x,y,z,t) \in \mathbb{R}^4 \mid x - z - t = 0 \; ; \; y + z = 0 \}$

• U

Bases:

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Bases de $U \equiv \{(1,0,1,1),(0,1,2,1)\}$

$\dim U = 2$

Ec. vectorial: $(x,y,z,t) = \lambda(1,0,1,1) + \alpha(1,-1,-1,0) + \mu(0,1,2,1)$

Ec. paramétrica: $\begin{cases} x = \lambda + \alpha \\ y = -\alpha + \mu \\ z = \lambda - \alpha + 2\mu \\ t = \lambda + \mu \end{cases}$

Ec. cartesiana:

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & x \\ 0 & -1 & 1 & y \\ 1 & -1 & 2 & z \\ 1 & 0 & 1 & t \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & x+y \\ 0 & 1 & -1 & -y \\ 0 & 0 & 0 & z-x-2y \\ 0 & 0 & 0 & t-x-y \end{array}\right)$$

$U \equiv \{(x,y,z,t) \in \mathbb{R}^4 / z - x - 2y = 0 \; ; \; t - x - y = 0\}$

• V:

Ec. cartesianas: $\{(x,y,z,t) \in \mathbb{R}^4 / x - z - t = 0 \; ; \; y + z = 0\}$

Ecu. paramétrica: $\begin{cases} x = \lambda + \mu \\ y = -\lambda \\ z = \lambda \\ t = \mu \end{cases}$

Ec. vectorial: $(x,y,z,t) = \lambda(1,-1,1,0) + \mu(1,0,0,1)$

Sistema Generador de $V = \{(1,-1,1,0),(1,0,0,1)\}$

Base:

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

Base de $V = \{(1,-1,1,0),(0,-1,1,0)\}$

$\dim V = 2$

• $U \cap V \equiv \begin{cases} x-z-t=0 \\ y+z=0 \\ t-x-2y=0 \\ t-x-y=0 \end{cases}$

$$\begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ -1 & -2 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$U \cap V : \begin{cases} x-z-t=0 \\ y+z=0 \\ -2y-t=0 \end{cases} \to$ E. ~~par~~ cartesiana

Ec paramétrica: $\begin{cases} x = 3\lambda \\ y = -\lambda \\ z = \lambda \\ t = 2\lambda \end{cases}$ SG y Base: $\{(3,-1,1,2)\}$

$\dim(U \cap V) = 1$

E vectorial: $(x,y,z,t) = \lambda(3,-1,1,2)$

• $U+V = \{(1,-1,1,0),(1,0,0,1),(1,0,1,1),(1,-1,-1,0),(0,1,2,1)\}$ $\dim(U+V)=3$ $B = \{(1,-1,0,0),(0,1,0,1),(0,0,1,0)\}$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Algebra Lineal   Propuestos Tema 2

1) a) $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3 , 2x_1 - x_2)$

$u = (x, y, z) \qquad v = (x', y', z')$

$f(u+v) = ((x+x') + (y+y') + (z+z') , 2(x+x') - (y-y')) =$

$= ((x+y+z) + (x'+y'+z'), (2x-y) + (2x'-y')) =$

$= f(u) + f(v) \qquad \Rightarrow$ Es aplicación lineal

b) $f(x_1, x_2, x_3) = (x_1^2 - x_2^2 , 2x_3 , 0)$

$u = (1,1,0) \qquad v = (0,1,1)$

$f(v+u) = f(1,2,1) = (1-4, 2, 0) = (-3, 2, 0)$

$f(u) + f(v) = (1-1, 0, 0) + (-1, 2, 0) \neq (-1, 2, 0)$

$\Rightarrow$ No es aplicación lineal

c) $f(x,y) = (x, y+2, x+y)$

$u = (1,1) \qquad v = (0,1)$

$f(u+v) = f(1,2) = (1, 4, 3)$

$f(u) + f(v) = (1,3,2) + (0,3,1) \neq (1,6,3)$

$\Rightarrow$ No es aplicación lineal.

d) $f(x,y) = (x+2y, 0, xy)$

$u = (1,2)$ $\quad v = (0,1)$

$f(1,2) = (5,0,2)$
$+$ $\quad = (7,0,2) \neq (7,0,3) = f(1,3)$
$f(0,1) = (2,0,0)$

2.- a) $f(1,0,0) = (1,2)$

$f(0,1,0) = (2,3)$

$f(0,0,1) = (-4,1)$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -4 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

b) $f(x_1, x_2, x_3) = (0,0)$

$$\left. \begin{array}{l} x_1 + 2x_2 - 4x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} x_1 + 14x_3 = 0 \\ -x_2 + 9x_3 = 0 \end{array} \right.$$ $\rightarrow$ E. cartesiana del ker f

Ecuación paramétrica ker f : $\left\{ \begin{array}{l} x_1 = -14\lambda \\ x_2 = 9\lambda \\ x_3 = \lambda \end{array} \right.$

$\dim \ker f = 1 \neq 0$
f no es inyectiva

$Im(f) = \mathcal{L}(f(1,0,0), f(0,1,0), f(0,0,1))$

$Im(f) = \mathcal{L}((1,2), (2,3), (-4,1))$

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \\ -4 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \left\{ \begin{array}{l} x_1 = t \\ x_2 = 2t + \lambda \end{array} \right.$$

Base $Im f = \{(1,0), (0,1)\}$

c) $f(1,-1,0)=(-1,-1)$

$f(-2,0,1)=(-6,-3)$

$f(0,0,-2)=(8,-2)$

$(-1,-1)=\alpha(-1,0)+\beta(-2,1)$

$(-6,-3)=\alpha(-1,0)+\beta(-2,1)$

$(8,-2)=\alpha(-1,0)+\beta(-2,1)$

$$Y=\begin{pmatrix} 3 & 12 & -4 \\ -1 & -3 & -2 \end{pmatrix} X$$

3:)

$$\begin{array}{ccc} \mathbb{R}^2 & \xrightarrow{f} & \mathbb{R}^2 \\ Bc(\mathbb{R}^2) & \xrightarrow{A} & Bc(\mathbb{R}^2) \\ P\uparrow & & \uparrow Q \\ B_1 & \longmapsto & B_2 \end{array}$$

$B=Q^{-1}AP$

$A=QBP^{-1}$

$B_1=\{(3\ 1),(1\ 1)\}$ $\quad B=\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$

$B_2=\{(0\ 2),(-1\ 1)\}$

$P=\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad Q=\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$

$(3\ 1)=\alpha_1(1\ 0)+\beta_1(0\ 1)$

$(1\ 1)=\alpha_1(1\ 0)+\beta_1(0\ 1)$

$f(3\ 1)=-2(0,2)+0(-1,1)=(0,-4)$

$f(1\ 1)=-0(0,2)+1(-1,1)=(-1,1)$

$$f(1,0)=\frac{f(3,1)-f(1,1)}{2}=\frac{(1,-5)}{2}=(1/2,-5/2)$$

$$f(0,1)=\frac{f(3,1)-3f(1,1)}{-2}=\frac{(0,-4)+(3,-3)}{-2}=\frac{(3,-7)}{-2}=(3/-2,7/2)$$

4- a) ker f $f(x,y,z) = (x+y+z, x+y, z)$

$\ker f := \{ (x,y,z) \in \mathbb{R}^3 \,/\, f(x,y,z) = 0 \}$

$\begin{cases} x+y+z=0 \\ x+y=0 \\ z=0 \end{cases} \overset{*}{\Rightarrow} \begin{cases} x+y=0 \\ z=0 \end{cases}$ Ec. cartesianas del ker f

$* \left(\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \to \left(\begin{array}{ccc:c} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)$

$\begin{cases} x+y=0 \\ z=0 \end{cases} \Rightarrow \begin{cases} x=-\lambda \\ y=\lambda \\ z=0 \end{cases}$ Ec. paramétrica del ker f

SG ker f: $\{(-1,1,0)\}$ Base ker f: $((1,-1,0))$

$\dim \ker f = 1 \Rightarrow$ No es inyectiva $\Rightarrow$ no es biyectiva.

Im f

$\mathrm{Im} f := \mathcal{L}\left(f(1,0,0), f(0,1,0), f(0,0,1) \right)$

$\mathrm{Im} f = \mathcal{L}\left((1,1,0), (1,1,0), (1,0,1) \right)$

$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & +1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$\mathrm{Im} f = \mathcal{L}\left((1,1,0), (0,1,-1) \right)$ $\dim \mathrm{Im} f = 2$

$\begin{cases} x=\lambda \\ y=\lambda+\mu \\ z=-\mu \end{cases}$ Ec. paramétricas de Im f

$\{ (x,y,z) \in \mathbb{R}^3 \,/\, x-y-z=0 \} \to$ Ec. cartesiana de Im f

b) $f(V)$ $V \equiv x_3 = 0$ $f(x,y,z) = (x+y+z\ ,\ x+y\ ,\ z)$

$V: \begin{cases} x = \lambda \\ y = \mu \\ z = 0 \end{cases}$ Base $V = \{(1,0,0),(0,1,0)\}$

$\mathcal{L}(f(V)) = \mathcal{L}\,(f(1,0,0)\ ,\ f(0,1,0))$

$= \mathcal{L}\,((1,1,0)\ ,\ (1,1,0))$

$= \mathcal{L}\,((1,1,0))$

c) $f(2,3,0)$ en $f(V)$

$f(2,3,0) = (2+3,\ 2+3,\ 0) = (5,5,0)$

$(5,5,0) = \alpha(1,1,0)$

$\alpha = 5$

d) $f^{-1}(3,2,1)$

$\left.\begin{aligned} x+y+z &= 3 \\ y+x &= 2 \\ z &= 1 \end{aligned}\right\}$ $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$\Rightarrow \begin{cases} x+y = 2 \\ z = 1 \end{cases}$ Ec. cartesianas de $f^{-1}(3,2,1)$

$\Rightarrow \begin{cases} x = 2-\lambda \\ y = \lambda \\ z = 1 \end{cases}$ E. paramétricas de $f^{-1}(3,2,1)$

6º/ $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = 0$$

$\lambda_1 = 4$ y $\lambda_2 = -1$

- $\lambda_1 = 4$ dimensión algebraica: 1 $d_1 = 1$
- $\lambda_2 = -1$ dimensión algebraica: 1 $d_2 = 1$

$\Rightarrow$ Es diagonalizable

$$(A - 4I) = \begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix} \qquad rg(A-4I) = 1$$

dimensión geométrica: $\underset{\substack{\uparrow \\ n}}{2} - \underset{\substack{\uparrow \\ rg}}{1} = 1$

$$(A - (-1)I) = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \qquad rg(A+I) = 1$$

dimensión geométrica: 1

$0 = \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ -y \end{pmatrix} \equiv 3x - 2y = 0 \qquad v_1 = (2/3, 1)$

$0 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \equiv x + y = 0 \qquad v_2 = (1, -1)$

$\Rightarrow$ Matriz de Pasa: $\begin{array}{c} \\ I \end{array} \begin{array}{cc} v_1 & v_2 \\ \end{array}$ $\begin{pmatrix} 2/3 & 1 \\ 1 & -1 \end{pmatrix}$

$\Rightarrow$ Matriz diagonalizada: $\begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$

$\Rightarrow$ Base de vectores propios: $\{(1,0), (0,1)\}$

$\Rightarrow$ $A = P^{-1} D P$

7:) $f(x,y,z) = (x\ ,\ ax+y\ ,\ x+y+2z)$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ a & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = (1-\lambda)^2(2-\lambda)$$

- $\lambda_1 = 1$ multiplicidad algebraica: 2 m. geometrica:
- $\lambda_2 = 2$ multiplicidad algebraica: 1 multiplicidad geometrica = 1

$$(A - 2\lambda) = \begin{pmatrix} -1 & 0 & 0 \\ a & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} x = 0 \\ y = 0 \\ z = \lambda \end{cases}$$

$U_2 = (0, 0, 1)$ $\quad rg(A - 2\lambda) = 2 \quad d_2 = 3 - 2 = 1$

$$(A - \lambda) = \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} ax = 0 \\ x+y+z = 0 \end{cases}$$

Si $a = 0 \Rightarrow d_1 = 3 - 1 = 2$ //

Tiene que ser $a=0$ y entonces las multiplicidades coincidiran y será diagonalizable.

8c/ $A = \begin{pmatrix} 1 & 0 & 0 \\ a & a & 0 \\ 2 & b & 2 \end{pmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ a & a-\lambda & 0 \\ 2 & b & 2-\lambda \end{vmatrix} = (1-\lambda)(a-\lambda)(2-\lambda)$$

- $\lambda_1 = 1$ $\quad \lambda_2 = a$ $\quad \lambda_3 = 2$
  $\alpha_1 = 1$ $\quad \alpha_2 = \cancel{4}\,1$ $\quad \alpha_3 = 1$ $\quad \rightarrow$ Si $a \neq 1$ y 2
  $d_1 = 1$ $\quad d_2 = 1$ $\quad d_3 = 1$ $\qquad$ b da igual.

- Si $a = 1$

  $\lambda_1 = 1$ $\quad \lambda_2 = 2$
  $\alpha_1 = 2$ $\quad \alpha_2 = 1$
  $d_1 = ?$ $\quad d_2 = 1$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & b & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x = 0 \\ by + z = 0 \end{cases}$$ Da igual el valor de b que no será diagonalizable

- Si $\cancel{a} = 2$

  $\lambda_1 = 1$ $\quad \lambda_2 = 2$
  $\alpha_1 = 1$ $\quad \alpha_2 = 2$
  $d_1 = 1$ $\quad d_2 = ?$

$$\begin{pmatrix} -1 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & b & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b & 0 \end{pmatrix}$$

Si $b = 0$

$\Rightarrow \begin{cases} x = 0 \\ y = \lambda \\ z = \mu \end{cases}$ y $d_2 = 2$ y es diagonalizable

Si $b \neq 0$ no lo será

9º/ $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

a) $|A - \lambda I| = \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} = (1-\lambda)(-5-\lambda)(4-\lambda) - 54 - 54 - 18(-5-\lambda)$

$+ (1-\lambda)(18) + 9(4-\lambda) = -\lambda^3 + 12\lambda + 16 = 0$

- $\lambda = 4 \quad \alpha_1 = 1 \quad d_1 = 1$
- $\lambda = -2 \quad \alpha_2 = +2 \quad d_2 = 2$

$(A - 4\lambda) = \begin{pmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} x + y - z = 0 \\ 2y - z = 0 \end{cases}$

$\Rightarrow \begin{cases} x = +\lambda \\ y = \lambda \\ z = 2\lambda \end{cases} \quad v_1 = (1,1,2)$

$d_1 = 3 - rg(A - 4\lambda) = 1$

$(A + 2\lambda) = \begin{pmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} x = \mu - \lambda \\ y = \mu \\ z = \lambda \end{cases}$

$d_2 = 3 - rg(A + 2\lambda) = 2 \quad B = \langle (-1,0,1), (0,1,1) \rangle$

$$\text{Paso} = \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \end{array}$$

b) $SG = \langle (1,1,2), (1,1,0), (-1,0,1) \rangle$

$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Base $: ((1,0,0), (0,1,0), (0,0,1))$ $\quad$ dim $= 3$

10/ $A = \begin{pmatrix} -1 & 2 & a \\ 0 & 1 & 2 \\ 0 & 4 & 3 \end{pmatrix}$

a) $(A-\lambda I) = \begin{pmatrix} -1-\lambda & 2 & a \\ 0 & 1-\lambda & 2 \\ 0 & 4 & 3-\lambda \end{pmatrix} = -(1+\lambda)(1-\lambda)(3-\lambda) + 8(1+\lambda) =$

$= (\lambda+1)^2(\lambda-5)$

• $\lambda_1 = 5$ $\quad \alpha_1 = 1$ $\quad d_1 = 1$

$(A-5I) = \begin{pmatrix} -6 & 2 & a \\ 0 & -4 & 2 \\ 0 & 0 & -0 \end{pmatrix}$

$d_1 = 3 - rg(A-5I) = 1$

• $\lambda_2 = -1$ $\quad \alpha_2 = 2$ $\quad d_2 = 2$

$(A+I) = \begin{pmatrix} 0 & 2 & a \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 2 & a \\ 0 & 0 & 0 \end{pmatrix} \overset{a=2}{=} \begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$d_2 = 2 = 3 - rg(A+I) =$

$\Rightarrow a = 2$

b) • $\begin{pmatrix} -6 & 2 & 2 \\ 0 & 4 & -2 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -6 & 2 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -6 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} +3 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} 3x - z = 0 \\ 2y - z = 0 \end{cases} \Rightarrow \begin{cases} x = 1/3\,\lambda \\ y = \frac{1}{2}\lambda \\ z = \lambda \end{cases}$ $\quad v_1 = (\frac{1}{3}, \frac{1}{2}, 1)$

• $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \{ y + z = 0 \Rightarrow \begin{cases} x = \lambda \\ y = \mu \\ z = -\mu \end{cases}$ $\quad v_2 = (1, 0, 0)$

$v_3 = (0, 1, -1)$

Base = $\{(1,0,0), (0,1,0), (0,0,1)$

10°/ c) $D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

d) $A = P^{-1} D P$

$P = \begin{pmatrix} \frac{1}{3} & 1 & 0 \\ 1/2 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

e) $A^6 = AAAAAA = P^{-1}DPP^{-1}DPP^{-1}DPP^{-1}DPP^{-1}DPP^{-1}DP =$

$= P^{-1} D^6 P$

$D^6 = \begin{pmatrix} 5^6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

11)

$$\begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix} \qquad a_0 = 1 \quad a_1 = 1 \quad , a_{200} = ?$$

$$\begin{pmatrix} a_2 \\ a_1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} = A \begin{pmatrix} a_1 \\ a_0 \end{pmatrix}$$

$$\begin{pmatrix} a_3 \\ a_2 \end{pmatrix} = A \begin{pmatrix} a_2 \\ a_1 \end{pmatrix} = A^2 \begin{pmatrix} a_1 \\ a_0 \end{pmatrix}$$

$$\vdots$$

$$\begin{pmatrix} a_{200} \\ a_{199} \end{pmatrix} = A^{199} \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} \qquad \Rightarrow \quad A^n = P D^n P^{-1}$$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 1 & -\lambda \end{vmatrix} = -\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda + 1)(\lambda - 2) = 0$$

$$(A + I) = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x = 1 \\ y = -1 \end{cases} \qquad v_1 = (1, -1)$$

$$\lambda_1 = -1$$

$$(A - 2I) = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x = 2\lambda \\ y = \lambda \end{cases} \qquad v_2 = (2, 1)$$

$$\lambda_2 = 2$$

$$D = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \qquad |P| = 3 \qquad P^T = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \quad \mathrm{adj}(P^T) = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} 1/3 & -2/3 \\ 1/3 & 1/3 \end{pmatrix}$$

$$D^2 = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$D^3 = \begin{pmatrix} +1 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 2^3 \end{pmatrix}$$

$$D^n = \begin{pmatrix} (-1)^n & 0 \\ 0 & 2^n \end{pmatrix}$$

$$D^{199} = \begin{pmatrix} -1 & 0 \\ 0 & 2^{199} \end{pmatrix}$$

$$A^{199} = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2^{199} \end{pmatrix} \begin{pmatrix} 1/3 & -2/3 \\ 1/3 & 1/3 \end{pmatrix}$$

$$A^{199} = \begin{pmatrix} -1 & 2^{200} \\ 1 & 2^{199} \end{pmatrix} \begin{pmatrix} 1/3 & -2/3 \\ 1/3 & 1/3 \end{pmatrix}$$

3º/ $f(3,1) = (-2)(0,2) + 0(-1,1) = (0,-4)$

$f(1,1) = (-1,-1)$

$Y = AX$

$\begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\quad$ $\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

12º/

$A = \begin{pmatrix} a-\lambda & b & c \\ d & e-\lambda & f \\ g & h & I-\lambda \end{pmatrix}$ $\quad$ $V_1 = \{(-1,1,0)\ (-1,0,1)\}$ $\lambda_1 = 1$

$v_2 = \{(1,1,1)\}$ $\lambda_2 = 1/2$

$\begin{pmatrix} a-1 & b & c \\ d & e-1 & f \\ g & h & I-1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} b-a = -1 \\ e-d = 1 \\ h-g = 0 \end{cases} \Rightarrow h = g$

$\begin{pmatrix} a-1 & b & c \\ d & e-1 & f \\ g & h & I-1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} c-a = -1 \\ f-d = 0 \\ I-g = 1 \end{cases} \Rightarrow f = d$

$\Rightarrow \begin{cases} h = g \\ f = d \\ b = c \\ e-1 = d \\ b-a = -1 \\ I-g = 1 \\ c-a = -1 \end{cases}$

$\begin{pmatrix} a-1/2 & b & c \\ d & e-1/2 & f \\ g & h & I-1/2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} a+b+c = 1/2 \\ d+e+f = 1/2 \\ g+h+I = 1/2 \end{cases}$

$d + d+1 + d = 1/2$ $\quad \boxed{f = -1/6}$

$3d = -1/2$ $\quad \boxed{e = 5/6}$

$\boxed{d = -1/6}$

$a + a-1 + a-1 = 1/2$ $\quad \boxed{b = -1/6}$

$3a = 5/2$ $\quad \boxed{c = -1/6}$

$\boxed{a = 5/6}$

$g+g+g+1 = 1/2$ $\quad \boxed{h = -1/6}$

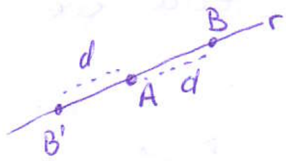
$3g = -1/2$ $\quad \boxed{I = 5/6}$

$\boxed{g = -1/6}$

$\boxed{A = \begin{pmatrix} 5/6 & -1/6 & -1/6 \\ -1/6 & 5/6 & -1/6 \\ -1/6 & -1/6 & 5/6 \end{pmatrix}}$

Alberto Delgado Expósito Algebra Lineal Tema 3

1:
$$r \equiv \begin{cases} x+y-z=2 \\ x-y+z=0 \end{cases} \qquad A=(1,1,0)$$



$d = 3\sqrt{2}$

$B\,(a,b,c) \qquad \overline{AB}\,(1-a,1-b,c)$

$$\begin{cases} \|\overline{AB}\| = 3\sqrt{2} \\ a+b-c=2 \\ a-b+c=0 \end{cases} \rightarrow \boxed{\begin{matrix} a=1 & b=-2 & c=-3 \\ a=1 & b=4 & c=3 \end{matrix}}$$

2: $P(3,2,1) \quad Q(3,1,-5) \qquad \pi: 6x+7y+2z=10 \qquad PQ=(0,1,6)$

$$\pi: \begin{cases} x=\lambda \\ y=\mu \\ z=5-3\lambda-\frac{7}{2}\mu \end{cases}$$

$v_{\pi_1} = (1,0,-3)$

$v_{\pi_2} = (0,1,-7/2)$

$$\begin{vmatrix} \vec{i}' & \vec{j}' & \vec{k}' \\ 1 & 0 & -3 \\ 0 & 1 & -7/2 \end{vmatrix} = 3\vec{i}' + \frac{7}{2}\vec{j}' + \vec{k}'$$

$v_T = (3, 7/2, 1)$

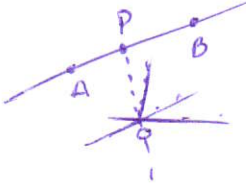
$$\boxed{\pi' = \begin{cases} x = 3 + + 3\mu \\ y = 2 + \lambda + 7/2\,\mu \\ z = 1 + 6\lambda + \mu \end{cases}}$$

3:

4:/ A (1, 0, 1)   B (2, 1, 3)

a) $d(AB, O)$ AB ≡ recta que pasa por A y B.

$\overline{AB}$ $(-1,-1,-2)$ $v(-1,-1,-2)$



$AB \equiv \begin{cases} x = 1-\lambda \\ y = -\lambda \\ z = 1-2\lambda \end{cases}$

$P(1-\lambda, -\lambda, 1-2\lambda)$ $\overline{PO}$ $(1-\lambda, -\lambda, 1-2\lambda)$

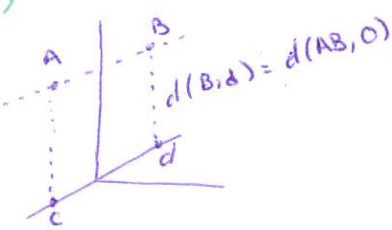
$\langle \overline{PO}, \overline{AB} \rangle = 0$ $P(1/2, -1/2, 0)$ $r_T \begin{cases} x = 1/2 + 1/2\lambda \\ y = -1/2 - 1/2\lambda \\ z = 0 \end{cases}$

$-1 + \lambda + \lambda - 2 + 4\lambda = 0$ $\overline{PO}$ $(1/2, -1/2, 0)$

$\boxed{\lambda = 1/2}$

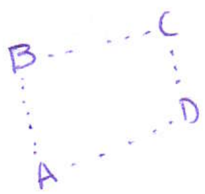
$d((AB), O) = \|\overline{PO}\| = \sqrt{1/4 + 1/4} = 1/\sqrt{2}$ //

b)



$A_p = d(AB, O) \cdot d(A,B) = \frac{1}{\sqrt{2}} \cdot \sqrt{6} = \sqrt{3}$

5:/ A(1, 0, 1)   B (3, 1, 4)   C (0, 2, a)   D (-2, 1, 6)



$\overline{AB} = (-2, -1, -3)$ $\overline{CD}$ $(2, 1, 3)$

$\overline{BC} = (3, -1, -5)$ $\overline{DA}$ $(-3, +1, 5)$

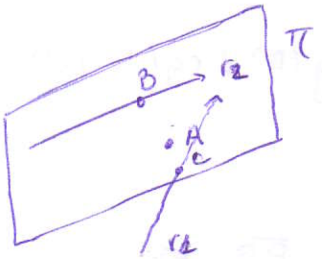
$d(A,B) = d(C,D) = \sqrt{14}$ $V_p = 2\sqrt{490}$

$A_B = \sqrt{490}$

$d(d, a) = d(B,C) = \sqrt{35}$

6: $r_1 \equiv \begin{cases} x - y - 2z = 2 \\ 3x - y = 1 \end{cases}$ $\quad r_2 \equiv \begin{cases} x = t \\ y = 1 + 2t \\ z = 0 \end{cases}$

a) Recta que pasa por $A(1,0,1)$ y r_1 y r_2



π contiene a r_2 y a

$r_1 \equiv \begin{cases} x = \lambda \\ y = 3\lambda - 1 \\ z = \frac{1}{2} + \lambda \end{cases}$

Elegimos un punto $B(0,1,0) \in r_2$ y $\vec{v}_{r_2}(1,2,0)$

Hacemos $\vec{AB}(1,-1,1)$

$$\boxed{\pi} = \begin{vmatrix} x-1 & y & z-1 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = \boxed{-2x + y + 3z = 1}$$

Donde corta π a r_1 en $C(\lambda, 3\lambda - 1, 1/2 + \lambda)$

$$-2(\lambda) + (3\lambda - 1) + \frac{3}{2} + 3\lambda = 1$$

$$-2\lambda + 3\lambda + 3\lambda - 1 + \frac{3}{2} = 1$$

$$4\lambda = 2 - \frac{3}{2}$$

$$4\lambda = \frac{1}{2}$$

$$\lambda = 1/8 \qquad \Rightarrow \quad C(1/8, 3/8 - 1, 1/2 + 1/8)$$

$$\boxed{C(1/8, -5/8, 10/16)}$$

recta que pasa por A, r_1 y r_2, es la que pase por A, B y C.

$r_{12A} \equiv \begin{cases} x = 1/8 + \lambda \\ y = -5/8 - \lambda \\ z = 10/16 + \lambda \end{cases}$

b) $r_1 \equiv \begin{cases} x - y - 2z = 2 \\ 3x - y = 1 \end{cases}$ $\qquad r_2 \equiv \begin{cases} y - 2x = 1 \\ z = 0 \end{cases}$

$$\begin{pmatrix} 1 & -1 & -2 & 2 \\ 3 & -1 & 0 & 1 \\ -2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{array}\right) 3 = rg(A) \neq rg(A^A) = 4$$

Se cruzan, veamos que puntos $P \in r_1$ y $Q \in r_2$, $\overline{PQ}$ es perpendicular a v_{r_1} y v_{r_2}.

$P(\lambda, 3\lambda - 1, \lambda + 1/2) \qquad Q(t, 1+2t, 0)$

$\overline{PQ}(\lambda - t, 3\lambda - 2t - 2, \lambda + 1/2)$

$\langle \overline{PQ}, v_{r_1} \rangle = 0$ Se hallan λ y t y $r_p \equiv \begin{cases} x = \\ y = \\ z = \end{cases} \begin{pmatrix} P \end{pmatrix} + \begin{pmatrix} \overline{PQ} \end{pmatrix} \lambda$

$\langle \overline{PQ}, v_{r_2} \rangle = 0$

c) $d(P, Q) = \|\overline{PQ}\|$

7/ a) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 1 + 1 + \lambda + \lambda + \lambda = 0$$

$$-\lambda^3 + 3\lambda + 2 = 0$$

$$(\lambda - 2)(\lambda + 1)^2 = 0$$

$\lambda_1 = 2 \quad \alpha_1 = 1 \quad d_1 = 1$

$\lambda_2 = -1 \quad \alpha_2 = 2 \quad d_2 = 2 \to$ Simetrica

$$(A + I) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x = -\lambda - \mu \\ y = \lambda \\ z = \mu \end{cases} \quad \{ x + y + z = 0$$

$v_1 = (-1, 1, 0) \qquad v_2 = (-1, 0, 1)$

$$(A - 2I) = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x - z = 0 \\ y - z = 0 \end{cases} \to \begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$$

$v_3 = (1, 1, 1)$

$\langle v_1, v_2 \rangle = 1$ Como no es ortogonal, usaremos GS

$e_1 = v_1 = (-1, 1, 0)$

$$e_2 = v_2 - \frac{\langle v_2, e_1 \rangle}{\langle e_1, e_1 \rangle} e_1 = (-1, 0, 1) - \frac{\langle (-1,0,1)(-1,1,0) \rangle}{\langle (-1,1,0)(-1,1,0) \rangle} \cdot (-1, 1, 0) =$$

$$= (-1, 0, 1) - \frac{1}{2}(-1, 1, 0) = \left(-\frac{1}{2}, -\frac{1}{2}, 1\right)$$

$$e_3 = v_3 - \frac{\langle v_3, e_1 \rangle}{\langle e_1, e_1 \rangle} e_1 - \frac{\langle v_3, e_2 \rangle}{\langle e_2, e_2 \rangle} e_2 = (1,1,1) - \frac{\langle (1,1,1) \cdot (-1/2, -1/2, 1) \rangle}{\langle (-1/2, -1/2, 1)(-1/2, -1/2, 1) \rangle}$$

$$(-1/2, -1/2, 1) - \frac{\langle (1,1,1)(-1,1,0) \rangle}{\langle (-1,1,0)(-1,1,0) \rangle} (-1,1,0) = (1,1,1) - 0 - 0 = (1,1,1)$$

$e_1 = (-1, 1, 0) \qquad e_2 = (-1/2, -1/2, 1) \qquad e_3 = (1, 1, 1)$

$$X = \frac{e_1}{\|e_1\|} = \frac{(-1,1,0)}{\sqrt{2}} = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$$

$$Y = \frac{e_2}{\|e_2\|} = \frac{(-1/2,-1/2,1)}{3/2} = (-1/3, -1/3, 2/3)$$

$$Z = \frac{e_3}{\|e_3\|} = \frac{(1,1,1)}{\sqrt{3}} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$$

$$P = \begin{pmatrix} -1/\sqrt{2} & -1/3 & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/3 & 1/\sqrt{3} \\ 0 & 2/3 & 1/\sqrt{3} \end{pmatrix} \qquad D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

b) $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -\lambda^3 + \lambda + \lambda = 0$$

$\lambda_1 = 0 \quad \alpha_1 = 1 \quad d_1 = 1$

$\lambda_2 = \sqrt{2} \quad \alpha_2 = 1 \quad d_2 = 1$

$\lambda_3 = -\sqrt{2} \quad \alpha_3 = 1 \quad d_3 = 1$

$v_3 = (1, -1/\sqrt{2}, -1/\sqrt{2})$ $\qquad X = (0, 1/\sqrt{2}, -1/\sqrt{2})$

$v_1 = (0, 1, -1)$ $\qquad Y = (1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2})$

$v_2 = (1, 1/\sqrt{2}, 1/\sqrt{2})$ $\qquad Z = (1/\sqrt{2}, -1/2, -1/2)$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix} \qquad P = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & 1/2 & -1/2 \end{pmatrix}$$

8)
$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & -1 & -1 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} = -\lambda^3 + 3\lambda^2 - 4 = 0$$

$$(\lambda+1)(\lambda-2)^2 = 0$$

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{cases} x-z=0 \\ y-z=0 \end{cases} \quad v_1 = (1,1,1)$$

$$\begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{cases} x = -\lambda-\mu \\ y = \mu \\ z = \lambda \end{cases} \quad \begin{matrix} v_2 = (-1,0,1) \\ v_3 = (-1,1,0) \end{matrix}$$

$$X = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$$

$$Y = (-1/\sqrt{2}, 0, 1/\sqrt{2})$$

$$Z = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$$

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$P' = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{2} & 0 \end{pmatrix}$$

9.-/ a) (a,b,c) tal que $\langle (a,b,c)(1,2,1)\rangle = 0$

$\langle (a,b,c)(0,-1,1)\rangle = 0$

$|(a,b,c)| = 1$

$\cancel{\langle (a,b,c)(a,b,c)\rangle = 0}$

$\langle (a,b,c)(1,2,1)\rangle = a+2b+c=0$

$\langle (a,b,c)(0,-1,1)\rangle = -b+c=0$

$\|(a,b,c)\| = a^2+b^2+c^2 = 1$

$$\begin{cases} a^2+b^2+c^2 = 1 \\ a+2b+c=0 \rightarrow a=-3c \\ -b+c=0 \rightarrow b=c \end{cases}$$

$9c^2+c^2+c^2=1$

$11c^2-1=0$

$c=\pm\sqrt{\frac{1}{11}}$ $\quad b=\pm\sqrt{\frac{1}{11}}$ $\quad a=\mp 3\sqrt{\frac{1}{11}}$

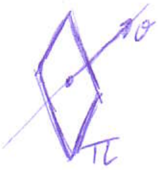
$\left(\mp 3\sqrt{\frac{1}{11}},\ \pm\sqrt{\frac{1}{11}},\ \pm\sqrt{\frac{1}{11}}\right) = (a,b,c)$

10:/ $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$

b) $\langle t+3, 2t+4\rangle = \int_0^1 (t+3)(2t+4)\,dt = \int_0^1 2t^2+4t+6t+12\,dt =$

$= \int_0^1 2t^2+10t+12\,dt = \frac{2t^3}{3}+5t^2+12t\Big]_0^1 = \frac{2}{3}+5+12 = \frac{53}{3}$ //

13c) $v(1,2,2)$ $\quad \alpha = \frac{\pi}{6}$



$\pi \equiv x + 2y + 2z = 0 \implies \pi \equiv \begin{cases} x = -2\lambda - 2\mu \\ y = \lambda \\ z = \mu \end{cases}$ $\quad \{(-2,0,1),(-2,1,0)\}$

$rg \begin{pmatrix} -2 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix} = 2$

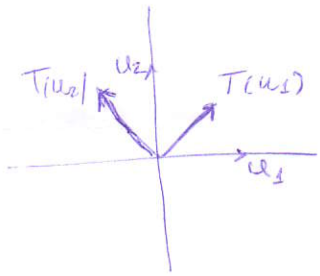
Base $\pi = \{(-2,0,1),(-2,1,0)\}$ No son ortogonales, haremos G.S.

$e_1 = (-2,0,1)$

$e_2 = (-2,1,0) - \frac{4}{5}(-2,0,1) = (-2/5, 1, -4/5) = (-2,5,-4)$

Normalizamos:

$u_1 = (-2/\sqrt{5}, 0, 1/\sqrt{5})$ $\quad u_2 = (-2/\sqrt{45}, 5/\sqrt{45}, -4/\sqrt{45})$ $\quad u_3 = (1/3, 2/3, 2/3)$



Respecto $B = \{u_1, u_2, u_3\}$

$T(u_1) = (\cos\frac{\pi}{6}, \operatorname{sen}\frac{\pi}{6}, 0)$ $\quad T(u_2) = (-\operatorname{sen}\pi/6, \cos\pi/6, 0)$

$T(u_3) = (0,0,1)$

$$A = \begin{pmatrix} \cos\pi/6 & -\operatorname{sen}\pi/6 & 0 \\ \operatorname{sen}\pi/6 & \cos\pi/6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{T} & \mathbb{R}^3 \\ B & \xrightarrow{A} & B \\ P\uparrow & & \uparrow P \\ B_c & \xrightarrow{D} & B_c \end{array}$$

14.)

$A(1,1)$ $B(1,2)$ $C(2,2)$

$A'(1,-1)$ $B'(2,0)$ $C'(3,-1)$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$	$\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$
$1 = a+b+e$ $-1 = c+d+f$	$2 = a+2b+e$ $0 = c+2d+f$
$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$	$3 = 2a+2b+e$ $-1 = 2c+2d+f$

$$\begin{cases} a+b+e = 1 \\ a+2b+e = 2 \\ 2a+2b+e = 3 \end{cases}$$

$b = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 1 & 3 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & +1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$a = 1$ $b = 1$ $e = -1$

$$\begin{cases} c+d+f = -1 \\ c+2d+f = 0 \\ 2c+2d+f = -1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 0 \\ 2 & 2 & 1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$c = -1$ $d = 1$ $f = -1$

$$\boxed{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix}}$$

$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ $\qquad AA^t = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$A^t = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$rg \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = 2$

15)

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \qquad A^t = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

Si fuera un mov $AA^t = I$, veamos si lo es:

$$\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ +\sqrt{3}/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Es un movimiento. Veamos cuál:

$$rg\,(A-I) = rg \begin{pmatrix} +1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} = 2$$

Tiene un pto fijo, es la rotación de centro en el pto fijo.

$$(A-I)\,X = -C$$

$$\begin{cases} -1/2\,x + \sqrt{3}/2\,y = -2 \\ -\frac{\sqrt{3}}{2}x + \frac{1}{2}y = -4 \end{cases} \to \begin{cases} +x + \sqrt{3}\,y = +2 \\ -\sqrt{3}\,x - y = -4 \end{cases}$$

$$+4y = +4 + 2\sqrt{3}$$

$$\boxed{\begin{array}{l} y = 1 + \frac{\sqrt{3}}{2} \\ x = \frac{1}{2} + \sqrt{3} \end{array}}$$

Pto al cual gira

16.)
$$y_1 = 1 + x_2$$
$$y_2 = \frac{3}{5}x_1 - \frac{4}{5}x_3 \quad \longrightarrow \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 3/5 & 0 & -4/5 \\ 4/5 & 0 & 3/5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$
$$y_3 = 2 + \frac{4}{5}x_1 + \frac{3}{5}x_3$$

- $T(0,0,0) = (1, 0, 2)$

- $AA^t = I$?

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 3/5 & 0 & -4/5 \\ 4/5 & 0 & 3/5 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 \\ 3/5 & 0 & -4/5 \\ 4/5 & 0 & 3/5 \end{pmatrix}\begin{pmatrix} 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \\ 0 & -4/5 & 3/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \\ 0 & -4/5 & 3/5 \end{pmatrix}$$

$$rg(A - I) = rg\begin{pmatrix} -1 & 1 & 0 \\ 3/5 & -1 & -4/5 \\ 4/5 & 0 & -2/5 \end{pmatrix} = rg\begin{pmatrix} -1 & 1 & 0 \\ 0 & -2/5 & -4/5 \\ 0 & 4/5 & -2/5 \end{pmatrix} = rg\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & -2 \\ 0 & 2 & -1 \end{pmatrix} =$$

$$= rg\begin{pmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 3 //$$ Tiene un punto fijo

$$(A - I)X = -C$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 3/5 & -1 & -4/5 \\ 4/5 & 0 & -2/5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$$

$$\begin{cases} -x_1 + x_2 = -1 \\ 3/5\, x_1 - x_2 - 4/5\, x_3 = 0 \\ 4/5\, x_1 - 2/5\, x_3 = -2 \end{cases} \longrightarrow \begin{cases} x_2 = -1 + x_1 \\ x_3 = \frac{(+2 + 4/5\, x_1)5}{2} \\ \frac{3}{5}x_1 + 1 - x_1 - 4 - \frac{8}{5}x_1 = 0 \end{cases}$$

$$-\frac{10}{5}x_1 = 3$$

$$\boxed{x_1 = -3/2} \quad \boxed{x_3 = 2}$$

El pto fijo es $(-3/2, -5/2, 2)$ $\boxed{x_2 = -5/2}$

17º) $P_1 = (0,0,0) \to (1,1,1)$

$P_2 = (1,0,0) \to (1,2,3)$

$P_3 = (1,1,0) \to (1,2,4)$

$P_4 = (1,1,1) \to (0,0,0)$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} j \\ k \\ l \end{pmatrix}$$

- $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} j \\ k \\ l \end{pmatrix} \Rightarrow \boxed{j = 1 \; ; \; k = 1 \; ; \; l = 1}$

- $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} 1 = a + 1 \\ 2 = d + 1 \\ 3 = g + 1 \end{matrix} \Rightarrow \boxed{\begin{matrix} a = 0 \\ d = +1 \\ g = 2 \end{matrix}}$

- $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 & b & c \\ 1 & e & f \\ 2 & h & i \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} 1 = b + 1 \\ 2 = 1 + e + 1 \\ 4 = 2 + h + 1 \end{matrix} \Rightarrow \boxed{\begin{matrix} b = 0 \\ e = 0 \\ h = 1 \end{matrix}}$

- $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & c \\ 1 & 0 & f \\ 2 & 1 & i \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} 0 = c + 1 \\ 0 = 1 + f + 1 \\ 0 = 2 + 1 + i + 1 \end{matrix} \Rightarrow \boxed{\begin{matrix} c = -1 \\ f = -2 \\ i = -4 \end{matrix}}$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -2 \\ 2 & 1 & -4 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad A^t = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{pmatrix}$$

Veamos si es un movimiento:

$$AA^t = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -2 \\ 2 & 1 & -4 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 5 & 10 \\ 4 & 10 & 21 \end{pmatrix} \neq I$$

No es un movimiento.

Veamos si tiene pto fijos: $(A-3)^2 X = -(A-3)C$

$\cancel{(A-I)X = -C}$

$\cancel{\begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} x_1 + x_3 = 1 \\ x_1 - x_2 - 2x_3 = -1 \\ 2x_1 + x_2 - 5x_3 = -1 \end{cases} \to \begin{cases} x_1 + x_3 = 1 \\ + x_2 + 3x_3 = + \\ + x_2 - 7x_3 = -1 \end{cases}}$

$$(A - 3) = \begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix}$$

$\cancel{(A-I)^2 = \begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix} \begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 6 \\ -6 & -1 & 11 \\ -11 & -6 & 24 \end{pmatrix}}$

$$\cancel{\begin{pmatrix} -1 & -1 & 6 \\ -6 & -1 & 11 \\ -11 & -6 & 21 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = -\begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$$

$$\cancel{\begin{pmatrix} -x_1 - x_2 + 6x_3 \\ -6x_1 - x_2 + 11x_3 \\ -11x_1 - 6x_2 + 21x_3 \end{pmatrix} = +\begin{pmatrix} +2 \\ +2 \\ +2 \end{pmatrix}}$$

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\begin{cases} x_1 + x_3 = 1 \\ x_1 - x_2 - 2x_3 = -1 \\ 2x_1 + x_2 - 5x_3 = -1 \end{cases} \rightarrow \begin{cases} x_1 + x_3 = 1 \\ +x_2 + 3x_3 = +2 \\ x_2 - 7x_3 = -3 \end{cases} \rightarrow \begin{cases} x_1 + x_3 = 1 \\ x_2 + 3x_3 = +2 \\ -10x_3 = -5 \end{cases}$$

$$\boxed{\begin{gathered} x_3 = 1/2 \\ x_2 = 1/2 \\ x_1 = 1/2 \end{gathered}}$$